
ALLOMETRIC EQUATIONS AND EXPANSION FACTORS FOR TROPICAL
DRY FOREST TREES OF EASTERN SINALOA, MEXICO

*Tropical and
Subtropical*

[ECUACIONES ALOMETRICAS Y FACTORES DE EXPANSIÓN PARA
ÁRBOLES DEL TRÓPICO SECO DE SINALOA, MÉXICO]

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SUMMARY

This research report aimed at estimating: (i) bole volume equations, (ii) aboveground biomass component equations, and (iii) biomass expansion factors, BEF, for aboveground biomass components for tropical dry trees of eastern Sinaloa, Mexico. Field work included measuring dasometric feature, dissecting and fresh-weighting trees from biomass components, and collecting samples for oven dry weight measurements. Bole volume and biomass component equations fitted an inventory data set to estimate biomass expansion factors at the plot scale. Results provided information for bole volume and biomass component equations. Mean biomass expansion factors (\pm sd) reported are 0.7854 (0.111), 0.873 (0.055) and 1.460 (0.022) for branches, bole and total aboveground biomass, respectively. Biomass expansion factors are dependent on mean stand diameter at breast height, dbh, and they do not distribute normally. Therefore the Weibull distribution was fitted to biomass expansion figures.

Key words: Bole volume, biomass components, Weibull, diameter dependent.

INTRODUCTION

Bole volume estimates are useful in forest inventory since the volume of timber is the basic management unit of forests. From forest volume, the allocation of several forest products can be derived such as poles for fences, sawn wood, pulp and paper, and plywood. Estimates of tree biomass are useful in assessing forest structure and condition (Chavé et al., 2003); forest productivity, carbon stocks and fluxes based on sequential changes in biomass; sequestration of carbon in biomass components; i.e., wood, leaves, and roots

RESUMEN

Se estimó: (i) ecuaciones de volumen del fuste, (ii) ecuaciones de componentes de biomasa y (iii) factores de expansión de biomasa, BEF, para componentes de biomasa aérea de bosques tropicales secos del este de Sinaloa, México. El trabajo de campo incluyó la medición de las variables dasométricas, la separación y pesado en fresco de los componentes de biomasa de los árboles y la colección de muestras para el secado en las estufas. Las ecuaciones de volumen del fuste y de componentes de biomasa se usaron con los datos del inventario para estimar los factores de expansión de biomasa a la escala del sitio. Los resultados muestran las ecuaciones de volumen fustal y de componentes de biomasa. Los factores de expansión de biomasa promedio (\pm DE) reportados son 0.7854 (0.111), 0.873 (0.055) y 1.460 (0.022) para ramas, fustes y biomasa total sobre el suelo, respectivamente. Los factores de expansión de biomasa son dependientes del diámetro promedio a la escala del sitio y no se distribuyen normalmente. Por esta razón, la distribución probabilística Weibull se ajustó a los factores de expansión de biomasa.

Palabras clave: Volumen fustal, componentes de biomasa, distribución Weibull, dependencia del diámetro.

and they can be used as an indicator of site productivity.

Several biomass-prediction equations are available for tropical tree species (Brown et al., 1989; Chambers et al., 2001; Chavé et al., 2005; Overman et al., 1994). However, species and mixture of species differ in allometry, wood density, and architecture, all of which can affect the relationship between the measurements taken during forest inventories and the biomass of individual trees. Therefore biomass equations are required for other forest ecosystems where there is little information. Equations developed from single

and mixture of species yield useful estimates for specific sites and for large-scale global and regional comparisons. These two types of equations are important in forest inventory data to which single or mixed-species equations are often applied.

The application of bole volume and biomass component equations in forest inventory data usually yields the biomass expansion factors, BEF, when biomass (Mg ha^{-1}) is divided by standing bole volume ($\text{m}^3 \text{ha}^{-1}$). With the use of BEF values, standing volumes are easily converted to biomass and carbon stocks. However, BEF figures are variable and they are always a function of stand age, stand mean diameter, and top height (Levy et al., 2004; Lehtonen et al., 2004; Gracia et al., 2004). Black et al. (2004) in an extensive study with inventory data sets of the USA and Ireland noted the need to develop local allometric equations with the aim to further expand to biomass factors.

In this study, we developed allometric equations for tree species of tropical deciduous forests of eastern Sinaloa, Mexico with the aim to be employed with forest inventory data sets to estimate biomass expansion factors.

MATERIAL AND METHODS

Study site

The research was conducted at the ejidos San Francisco Javier de Tinequis and Vado Hondo in the Mexican State of Sinaloa (Figure 1). The ejidos are located in the dry tropical forest of the lowlands of the Pacific Ocean with mean annual temperature and rainfall of 21°C and 700 mm, respectively. Soils in the lowlands of the Pacific Ocean are usually

characterized by Vertisols and Regosols. Vertisols are deep soils, which shrink and swell with changes in soil moisture content. They are usually red in color, with fine clay texture and extensively used in agriculture. When covered with tropical dry forests, Vertisols produce the largest trees. Regosols are shallow soils with medium, loam texture. They are well developed with a great amount of rocks and distribute in the slopes of the Sierra with different tropical dry forests.

Species

The research project comprised taper, volume and biomass measurements of 40 trees of six different species. All species are native to the studied forests, and all are species of economic importance since they are harvested for charcoal production, fence poles, fodder, furniture, etc.

The study was conducted in the Sinaloa's Dry Forest plant community, which covers an area of 29,900 square miles (i.e. 77,500 square kilometers) and stretch through three Mexican states, Sonora, Sinaloa and Nayarit, from the base of the Sierra Madre Occidental Mountains west to the Pacific Ocean. The climate of this area is subtropical, subhumid with a long dry season. In this climatic zone distributes the dry forest trees and columnar cacti, including the kapok tree, quiebrocha, and desert fern. Species commonly found according to a forest inventory are: *Lysiloma divaricata*, *Haematoxylon brasiletto*, *Bursera penicillatum*, *Erythrina guatemalensis*, *Guazuma ulmifolia*, *Ipomoea arborescens*, *Ceiba acuminata*, *Pithecellobium mangense*, *Rubus palmeri*. These species are also distributed widely in the tropical dry forests of America. From these common species only six of them were selected harvesting in total 40 trees for biomass measurements.



Figure 1. The Mexican State of Sinaloa in northwestern Mexico (Source: [http://images.google.com / imgres?](http://images.google.com/imgres?)).

Samples

The experimental design comprised the selection of 40 trees of different diameters at breast height, DBH, (Table 1). Trees were selected proportionally per diameter class until completing the number of trees used in this study. DBH was recorded using diameter tapes on standing trees. Trees were felled and diameters were measured at the base, 0.50, 1 m, 1.3 m, 2.0 m and every meter thereafter on felled trees, where top height was also measured.

DBH of each harvested species was greater than 5.2 cm; *Lysiloma divaricata* had the highest value, 32.6 cm. Diameters of trees were recorded using diameter tapes with a precision of 0.1 cm. Top height of trees was recorded using measurement tapes with a precision of 0.1 m. Top heights were measured directly on the main stem after trees were felled. Harvested trees were dissected into their component parts. At a minimum this comprised leaves and branches together and boles. Boles were logged to minimum commercial log size, 2.50 m, for further commercial use. All leaves and branches and logs per tree were fresh weighted. The total fresh weight of each component was obtained in the field using electronic balances and recorded to 1 g for material weighing <5 kg or to 10 g for heavier material. Samples of each component for each tree were selected fresh weighted and oven-dried (to constant weight at 70 °C). Dry weights were recorded to 0.1 g.

Data analysis

Data analyses were generated using SAS software (SAS Institute Inc., Cary, NC, USA). When analyzing the data, we first examined scatter plots of biomass components and DBH. In the case of values suspected of being erroneous, we checked field and laboratory data and all calculations. If we were confident that data had been taken and recorded properly, those data remained in the data set at least temporarily.

I fit the standard form of the allometric equation: $y_i = a(\text{DBH})^b$, where y_i = biomass of tree component i , DBH is the diameter at breast height, and a and b are statistical parameters for species and groups of species. This was accomplished by transforming the data to the natural logarithm; i.e., $\text{Ln}(y_i) = a + b(\text{Ln}(\text{DBH}))$. I then examined the fit of the data to the equation and reviewed for possible data-recording errors; any data point whose studentized residual (i.e., the ratio of the residual to its standard error) exceeded a value of ± 3 . In almost all cases, those data points were declared to be outliers and excluded from further analysis. After eliminating outliers, we again calculated the equation's parameters.

I first calculated the scaling factors for two sets of equations for each component of each species. Slopes and intercepts of all relationships were determined, as were the significance of differences among them, using PROC REG (SAS, 2000). This procedure provides the slope and intercept parameters and the equation does not require a correction factor that accounts for the presence of large biomass data used in when log transforming the data.

The bole volume equation was developed using both DBH and H, fitting the classical equation of Schumacher and Hall (1933) using the intrinsic linear equation; i.e., $\text{Ln}(V) = a + b_1(\text{Ln}(\text{DBH})) + b_2(\text{Ln}(H))$. Bole is defined in this study as the main stem of the tree from the soil surface to the place where the first large branch protrudes the stem.

Forest inventory data for 168 quadrats (20 m x 20 m) for the tropical dry forest of the ejido San Francisco Javier Tineaquis was available to fit allometric equations. Unfortunately, in forest inventory of tropical dry forests DBH is the only dasometric variable measured. Therefore, an equation to predict bole volume by using the sole variable DBH was developed; i.e., $\text{Ln}(V) = a + b(\text{Ln}(\text{DBH}))$.

Table 1. Sample size and range of diameters of trees harvested to determine stature–biomass relationships.

Species (Common Name)	N	Diameter at Breast Height (cm)			Dry Weight Range (kg)	
		Mean	Maximum	Minimum	Maximum	Minimum
<i>Acacia cochliacantha</i> (binolo)	10	14.1	22.2	5.2	80.3	1.8
<i>Bursera penicillatum</i> (copal)	5	13.3	22.1	6.3	61.2	3.4
<i>Lysiloma divaricata</i> (mauto)	10	14.8	32.6	5.6	234.7	3.8
<i>Ceiba acuminta</i> (pochote)	5	13.8	19.1	6.6	47.7	0.3
<i>Cochlospermum vitifolium</i> (rosa amarilla)	5	17.3	31.4	5.5	80.2	1.6
<i>Jatropha angustifolia</i> (tachinole)	5	18.2	31.8	9.8	188.5	8.3
Total/Promedio	40	15.2	26.5	6.5	115.4	3.2

Allometric equations calculated standing bole volume and biomass components (branch, bole and total aboveground) at the quadrat scale by adding the volume and biomass component of each tree within the quadrat. Equation [1] estimated biomass expansion factors on per hectare basis, as follows:

$$BEF = \frac{\text{Biomass Component (Mg ha}^{-1}\text{)}}{\text{Bole Volume (m}^3 \text{ ha}^{-1}\text{)}} = BEF \left[\frac{\text{Mg}}{\text{m}^3} \right] \quad [1]$$

Linear regression equations fitted well the relationship BEF vs Mean quadrat DBH and parameters for the equations are reported to calibrate BEF when it is required. In addition, normality tests were conducted on the BEF values. The Shapiro-Wilks, Kolmogorov-Smirnoff and Cramer von Mises tests were conducted on the BEF's using Proc Univariate in SAS. Since FEB's did not fitted well the normal distribution function an alternate density function, i.e. the Weibull fitted the data better. The Weibull density functions and its parameters are also reported in this paper. The probabilistic density function of three parameters, as pdf, is described in equation [2], as follows:

$$P_x(X) = \left(\frac{\alpha}{\beta} \right) \left(\frac{X - \varepsilon}{\beta} \right)^{\alpha-1} e^{-\left(\frac{X - \varepsilon}{\beta} \right)^\alpha} \quad [2]$$

and as cumulative density function (cdf) as follows:

$$P(x \leq X) = 1 - e^{-\left(\frac{x - \varepsilon}{\beta} \right)^\alpha} \quad [3]$$

Where $P_x(X)$ = the probability of the random variable x , β y ε = are the shape, scale and location parameters of the pdf, respectively.

Návar and Contreras (2000) developed computer programs to fit the Weibull distribution to any random variable by employing several methods of parameter estimation. In this report, I used the moments methodology of parameter estimation. Equations [4, 5 and 6] calculated parameters of the Weibull distribution. Haan (1986) reported that the skew coefficient (γ) is related to the shape parameter (α) by equation (3):

$$\gamma = \frac{\Gamma(1+3/\alpha) - 3\Gamma(1+2/\alpha)\Gamma(1+1/\alpha) + 2\Gamma^3(1+1/\alpha)}{[\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)]^{3/2}} \quad [3]$$

The shape parameter is iteratively fitted by estimating first the skew coefficient and solving model [3] to yield β and ε as defined below.

$$\beta = \left[\frac{\sigma^2}{\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)} \right]^{1/2} \quad [4]$$

$$\varepsilon = \mu - \beta\Gamma(1+1/\alpha) \quad [5]$$

Where: μ and σ are the average and standard deviation of the random variable, respectively, and $\Gamma(x)$ is the gamma function.

RESULTS AND DISCUSSION

The bole volume equation of Schumacher and Hall (1933) with parameters estimated by linear regression using a correction factor is: $\text{Ln}(v)/1.02829 = (-10.4631 + 1.89587*\text{Ln}(Dn) + 0.6693*\text{Ln}(H))$; $r^2=0.96$. The bole volume equation that uses only normal diameter as independent variable and a weighting factor is: $\text{Ln}(v)/1.0815 = (-10.1288 + 2.1732*\text{Ln}(Dn))$; $r^2 =0.90$. This equation is presented in graphical format in Figure 2.

Heteroscedasticity is noted in Figure 2, where the bole volume variation increases with the dimensions of normal diameter. The weighting factor of the bole volume equations provides some improvement on the bole volume estimates since data compression calculates an equation that underestimates large values. Allometric equations that predict biomass components are reported in Figure 3.

The allometric equation is a satisfactory predictor of biomass since the total variation explained by the relationships was above 66%, with an average of (\pm confidence interval) 74% (\pm 8%). The relationship is much stronger for tree with larger biomass weights (i.e., those having large biomass amounts in the bole or total aboveground biomass; $r^2 > 0.77$). The branch and leaves biomass components are always difficult to predict with the precision of bole or total aboveground biomass. Competition for light between neighboring trees may account for some of this variation since trees growing under a strong competition and attaining a dominant position recorded small branch and foliage biomass. On the other side, trees growing in open spaces tended to have widespread crowns and large biomass in branches and leaves in contrast to that of the bole.

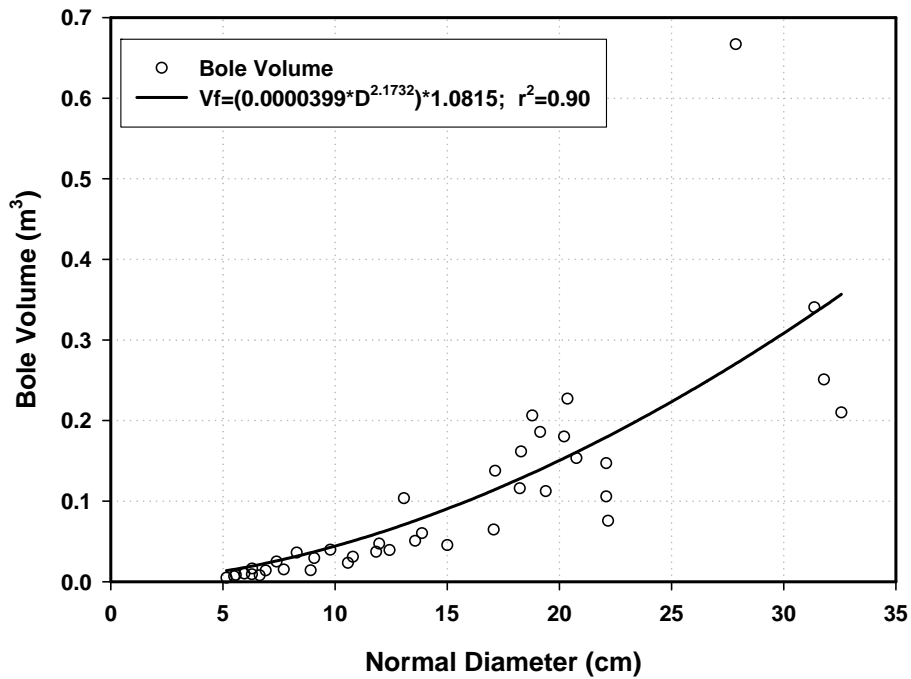


Figure 2. The bole volume equation for tropical deciduous dry trees of eastern Sinaloa, Mexico.

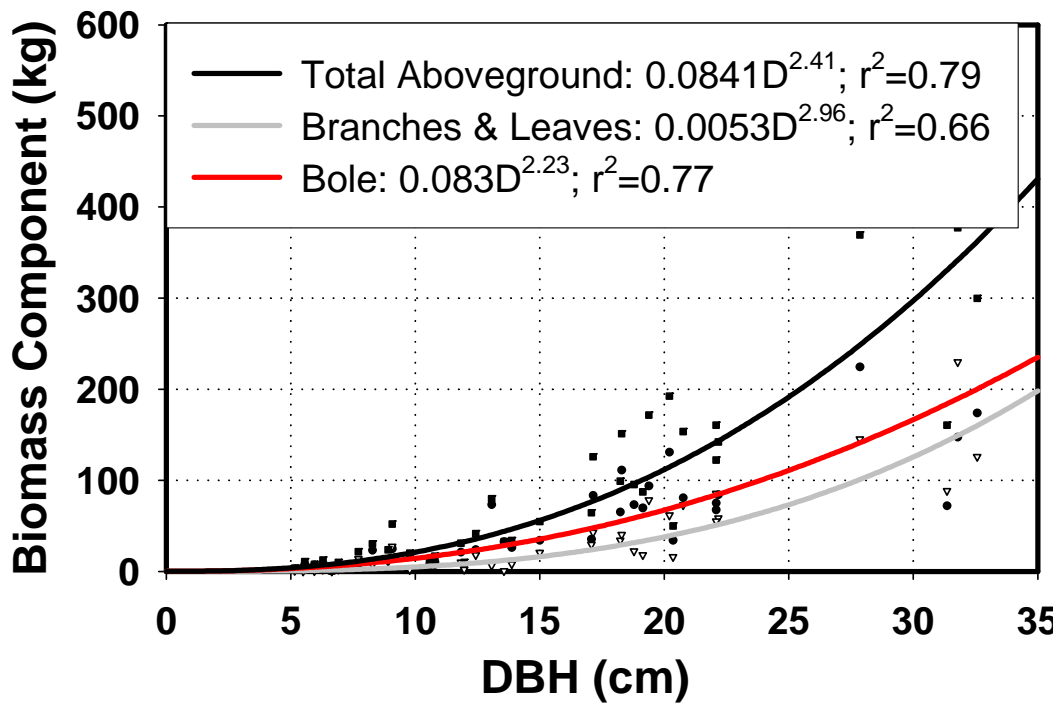


Figure 3. Biomass component equations for trees of tropical dry forests of eastern Sinaloa, Mexico.

Not surprisingly, the ability to predict the biomass of large woody components such as boles and total aboveground tends to be stronger than that of smaller, shorter-lived components such as branches and leaves. Branch and foliage biomass is susceptible to weather, herbivores, and inter-plant competition. In densely spaced forests, it is likely that intraspecific competition influenced crown geometry and therefore the heterogeneity of branch and leaf biomass from tree to tree. Because of the unusually large number of trees sampled, the equations are indicative of the degree of variation to be anticipated among trees of the same plant community: breakage, reiteration, herbivory, competition, and a number of other factors lead to the irregularity that is typical of species such as these.

The single metric most commonly used for tree allometry is diameter, as is evident in the review (65 species) of Ter-Mikaelian and Korzukhin (1997) and the synthesis (279 equations) compiled by Zianis and Mencuccini (2004). Statistical fits are generally good, enabling one to use locally developed equations with confidence for the stands from which they were derived. Tree height is more tedious to measure and may not explain more of the variance at the site where the data originated, but its incorporation has the advantage of increasing the equation's potential applicability to different sites (Ketterings et al., 2001).

The statistics and tests of normality for the biomass expansion factors are presented in Table 2. It was

earlier noted that the BEF's are dimensionally a stand density component of above biomass components. Therefore, when looking at the value of the BEF for boles, its mean 0.87 makes sense as a stand density component for tropical dry trees. This figure is dependent on the mixture of species. BEF for branches is a little bit smaller than the value for boles indicating that the branch component is also smaller than the bole component for the six species sampled.

The small standard error values, S_x , shows the consistency of the BEF for all biomass components. That is, the mean BEF values deviate no more than 1 % for branches and less than 0.50% for boles and total above ground biomass. The standard error values are smaller than deviations reported in other studies (Gracia et al., 2004; Lehtonen et al., 2004)

According to the Shapiro-Wilk normality tests all BEF are non normally distributed ($P=0.05$), unlike the K-S and Cramer - von Mises, which indicate that the BEF for branches is the only biomass component that distributes normally. However, they fit better the Weibull density function for branches, boles, and total above ground biomass since average deviations did not surpass 8.5 units (Figure 4). The Weibull distribution function parameters for branches are: $\alpha = 2.20$; $\beta = 0.81$ and $\epsilon = 0.55$; for boles $\alpha = 2.29$; $\beta = 0.88$ and $\epsilon = 0.75$; and for total above ground biomass $\alpha = 2.69$; $\beta = 1.46$ and $\epsilon = 1.40$.

Table 2. Statistics and normality tests for biomass expansion factors of tropical dry trees of eastern Sinaloa, Mexico.

	Mean	S_x	Shapiro-Wilk	K-S	Cramer-von Mises
BEF	0.8731	0.0043	0.97(0.0018)	0.08(0.010)	0.28(0.005)
BER	0.7854	0.0086	0.96(0.0004)	0.056(0.15)	0.08(0.21)
BET	1.4604	0.0017	0.97(0.0068)	0.08(0.010)	0.21(0.005)

BEF = Biomass expansion factor for boles, BER = Biomass expansion factor for branches and foliage, BET = Biomass expansion factor for total aboveground biomass, S_x = Standard error, K-S = Kolmogorov-Smirnoff.

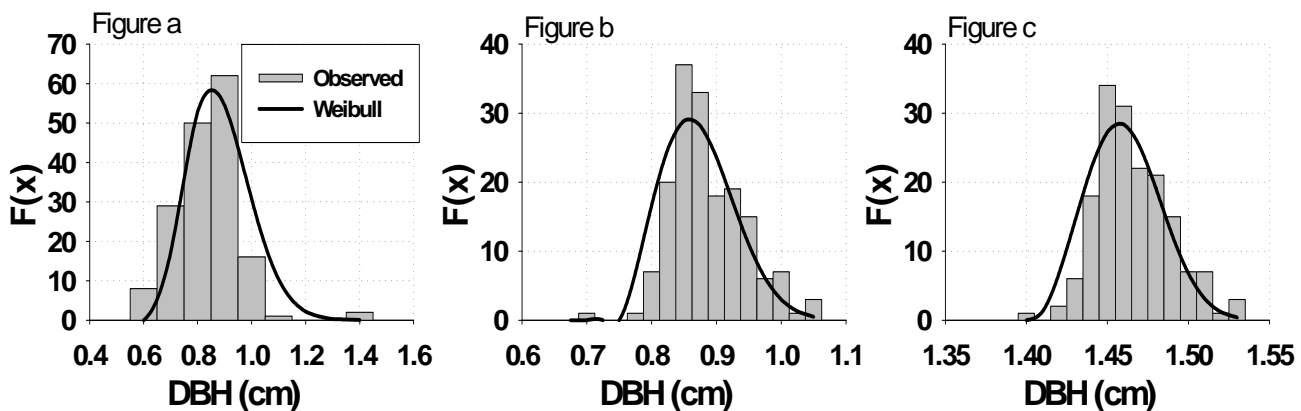


Figure 4. The Weibull distribution fitted to observed data of biomass expansion factors for a) branches and foliage, b) bole, and c) total aboveground biomass for tree species of tropical dry forests of eastern Sinaloa, Mexico.

Biomass expansion factors are tree-size dependent (Figure 5). The linear regression equations fitted well the BEF's tendencies against normal diameter for all biomass components. Bole and total above ground biomass expansion factors diminishes with an increment on mean stand normal diameter unlike the branch expansion factor which increases with mean stand normal diameter.

For branches, BEF's diminished in most species studied by Gracia et al., (2004); from 0.32 to 0.20 in trees of the species *Fagus sylvatica* with mean diameters of 7.5 and 57.5 cm; and from 0.49 to 0.33 for *Quercus ilex* with mean diameters of 7.5 cm and 32.5 cm. Lehtonen et al., (2004), on the other side, observed that BEF for total aboveground biomass increased in beech and pine trees but diminished in Oak and Spruce trees with incremental DBH's.

These relationships are consistent for boles and total above ground biomass but it is not consistent for branches. Should we have studied trees of larger dimensions the relationship probably curved in a typical quadratic relationship? Allometric equations for branches and boles cross each other when trees are 32 cm in DBH indicating that the branch component starts being larger than the bole component for trees of similar DBH's (Figure 3). A power relationship fitted better the branch biomass expansion factor as a

function of DBH, with a coefficient of determination, $r^2 = 0.86$ and an exponent of 0.74. The exponent is smaller than 1.0 and therefore the curve is desaccelerating trying to attain a steady state in the BEF. Similar findings are reported by Lehtonen et al., (2004) for beech and pine trees for branch biomass.

CONCLUSIONS

In this research report I present bole volume, biomass component equations and biomass expansion factors for tropical dry trees of eastern Sinaloa, Mexico. Mean values for biomass expansion factors for boles are quite consistent with wood densities of the studied tropical dry forests. Biomass expansion factors did not distribute normally and they fitted better a Weibull density function. It is recommended to use the mean BEF values to calculate biomass components with the sole standing volume variable. Before using BEF's, the bole must be correctly appraised and weighted by the mean stand DBH. In case there are uncertainties on where exactly to measure stem diameter to calculate bole volume, first it is recommended to estimate bole volume with the logarithmic equation and then simply apply the biomass expansion factor to calculate biomass components. Biomass components can also be calculated with the allometric equations recommended in this study. However, the procedure becomes tedious since the calculation is for all trees in the quadrat.

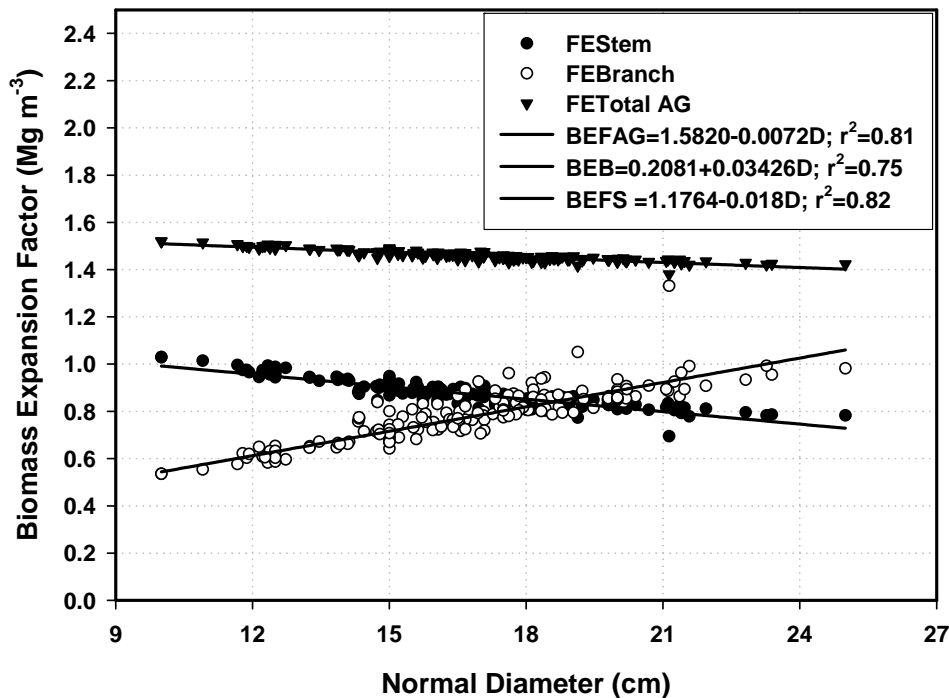


Figure 5. The linear dependency of biomass expansion factors on tree-size for tropical dry trees of eastern Sinaloa, Mexico.

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