FITTING AND TESTING ALLOMETRIC EQUATIONS FOR MEXICO'S SINALOAN TROPICAL DRY TREES AND FOREST INVENTORY PLOTS

[AJUSTE Y PRUEBA DE ECUACIONES ALOMETRICAS A LOS ARBOLES Y BOSQUES TROPICALES SECOS DE SINALOA, MEXICO]

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SUMMARY

Aboveground tree biomass (bole, branches and foliage), M, plays a key role in the conventional and sustainable management of forest communities. The standard approach to assess tree or plot M is harvesting trees, developing and fitting allometric equations to trees or forest inventory plot data. In the absence of local tree allometry, it is usually recommended to fit off site allometric equations to evaluate tree or plot M. This research aims: (a) to develop an updated on site allometric equation (b) to fit available off site allometric equations to destructively harvested trees and (c) to fit available allometric equations to plot M of Mexico's Sinaloan tropical dry forests to understand sources of inherent tree and plot M variability. Results showed that: (a) the improved on site allometric equation increases precision in contrast to the conventional biomass equation previously reported as well as to off site tree M equations, (b) off site allometry projects tree and plot M deviates by close to one order of magnitude. Two tested and recommended approaches to increase tree and plot M precision when fitting off site equations are: (i) to use all available tree allometric functions to come up with a mean equation or (ii) to calibrate off site equations by fitting new, local parameters that can be calculated using statistical programs.These options would eventually increase tree and plot M precision in regional evaluations.

Key words: biomass allometry; empirical; semi-empirical; theoretical allometric models.

RESUMEN

La biomasa aérea en pié (fustes, ramas y follaje), M, juega un papel importante en el manejo sustentable de las comunidades forestales. El método convencional que evalúa M de árboles o rodales forestales es por medio del desarrollo y aplicación de ecuaciones alométricas a los datos dasométricos de los árboles o del inventario forestal. En la ausencia de alometría desarrollada localmente, se recomienda generalmente el uso de ecuaciones alométricas universales, las cuales generalmente son desarrolladas fuera del sitio de interés. Esta investigación plantea por objetivos: (a) desarrollar una ecuación alométrica nueva y (b) ajustar ecuaciones alometricas disponibles a datos de M provenientes de árboles medidos en campo y (c) ajustar las ecuaciones a diferentes sitios ubicados en los bosques secos de Sinaloa, México para entender las fuentes de variación inherentes en la estimación de M al nivel del árbol y del sitio. Los resultados mostraron que: (a) la ecuación alométrica moderna desarrollada incrementa la precisión en contraste con las ecuaciones convencionales de biomasa, (b) la alometría desarrollada fuera del sitio proyecta valores de M que pueden desviarse por cerca del doble de las mediciones de M y (c) las evaluaciones al nivel del sitio también pueden desviarse por mas del doble cuando se usan ecuaciones desarrolladas fuera del sitio. Se probaron y recomendaron dos procedimientos para aumentar la precisión en la evaluación de M para árboles y sitios cuando se ajustan ecuaciones desarrolladas fuera del sitio: (i) usar las ecuaciones alométricas disponibles para estimar una promedio o (ii) calibrar las ecuaciones

individuales por el ajuste de parámetros locales, los cuales pueden ser calculados con el uso de programas estadísticos. Estas opciones incrementan la precisión en las evaluaciones de M de árboles y sitios y eventualmente de las selvas tropicales secas del norte de México.

Palabras clave: ecuaciones de biomasa; modelos empíricos; semi-empíricos, teóricos.

INTRODUCTION

The development and fitting of allometric equations is the standard methodology for estimating tree and plot aboveground biomass (Brown et al., 1989; Brown, 1997; Chavé et al., 2003; 2005; 2006; Návar, 2009a; 2010). Aboveground biomass assessments are critical for the evaluation of the amount of bio-energy contained in biomass as a partial alternative to fossil fuels in the clean and sustainable production of energy (Mckendry, 2002). Interest also centers on global environmental issues because forest ecosystems contribute to the global carbon cycle, and aid in mitigating the effects of climate change (Canadell and Rapauch, 2008). Aboveground correlates well with belowground biomass root stocks as well as with the litter and necro-mass stocks, hence evaluations are also important for the estimation of other biomass components (Cairns *et al.*, 1997; Mokany *et al*., 2006). The assessment of aboveground biomass also helps in understanding the resource allocation theories among tree organs (West *et al.*, 1999).

Biomass equations are classified as empirical, semi-empirical and process theoretical models (Návar, 2010). The most common empirical allometric equation reported in the scientific literature (Ter Mikaelian and Korzukhin, 1997; Jenkins et al., 2003; Zianis and Mencuccini, 2004; Návar, 2009b) is the conventional logarithmic model where M is estimated as a log linear function of diameter at breast height, D, with the scaling coefficients *a* and *B*. New updated empirical allometric equations contain the wood specific gravity value, ρ_w, and canopy height, H, in addition to diameter at breast height as exogenous independent variables (Chavé et al., 2005; Návar, 2009a). A restrictive model was proposed by Zianis and Mencuccini (2004) in which a small number of trees with the smallest diameter are harvested and fitted to reported biomass equations. A fully theoretical non-destructive procedure was derived using the theory of fractals (West et al., 1999), hereafter called the WBE model, where the main assumption is that D is related to M by $M \infty D^{8/3}$, pointing that the scaling exponent B_{WBE} equals 8/3 = 2.67 (West et al., 1999a; 1999b; Enquist et al., 1998). Preliminary reports stress that the WBE technique requires further refinement before can be recommended as a non-destructive tree M evaluation method because the scaling exponent *B* has been found to be smaller than 2.67 (Zianis and Mencuccini, 2004, Pilli et al., 2006; Návar, 2009a).

Semi-empirical models are becoming common in the scientific literature (Návar, 2010; 2011; 2012). Návar (2012) proposed and tested a semi-empirical non-destructive, and flexible procedure that uses shape-dimensional analysis coupled with fractal theory. The *B* slope coefficient is calculated with the timber volume, V, = f(D, H) and canopy height, H = f(D) functions and the *a*-intercept value was estimated by an empirical equation that takes advantage of the good statistical relationship between the scalar coefficients (*a* and *B*) reported in most meta-analysis allometric studies (Zianis and Mencuccini, 2004; Návar, 2009a,b; Návar-Cháidez, 2009). The methodology was tested for several Mexican temperate forest communities of northern Mexico and resulted in the precision given by most conventional biomass equations. Although better M assessments are provided by the Návar’s et al., (2013) non-destructive model than by the restrictive models, the method is complicated since it requires several local allometric relations to derive appropriate scalar coefficients. Návar (2013) proposed the population *B*-scalar exponent equals 2.38 by observing that this coefficient was the mean value of several biomass equation meta-analysis studies. This author went further by proposing the *a*-scalar intercept should then be a function of the wood specific gravity, ρ_w, to be consistent with both the theoretical and the conventional allometric models. This author developed regression equations for North American temperate as well as for American tropical trees to estimate the *a*-scalar intercept as a function of ρ_w and this model provided a good fit for the North American clusters of tree species proposed by Jenkins et al. (2003).

In spite of this brief literature review, several issues remain poorly understood on the biomass allometry of tropical forests. The following questions should be properly addressed before developing new or fitting available off site allometric equations. (a) Which off site allometric equation better suits tree and plot M for Mexico’s Sinaloan tropical dry forests? (b) What are the major inherent error sources when estimating tree and plot M with on and off site allometric equations? And (c) how does on and off site equations perform at the plot scale using forest inventory data? This study set as objectives: a) to develop an updated, modern on site allometric equation that can be broadly employed in Mexican western tropical dry forest M assessments; b) to contrast the updated on site to the off site allometric equations in order to understand sources of inherent variability; and (c) to fit available allometric equations to a forest inventory plots distributed at two ejdidos within the study area. These questions and objectives were solved with a destructively harvested tree data sample composed of 39 trees of six species and 390-0.1 ha rectangular (20x50 m) forest inventory plots distributed at two ejidos in the Mexico's Sinaloan tropical dry forests.

MATERIALS AND METHODS

Allometric theory

Biomass equations can be classified as empirical, semi-empirical and process models (Návar, 2010). Empirical equations often fit parameters by least square techniques in regression analysis. Theoretical models physically parameterize equations that convey meaningful biomass descriptions. Semi-empirical models, as the theoretical ones, are non-destructive techniques that require of both physically and statistically described parameters (Návar, 2010).

The WBE theoretical model. A fully, theoretical non-destructive model proposed by West et al. (1999a) was developed using the theory of fractals, WBE. This model applies to natural occurring networks that carry sustaining fluids in organisms, in which each small part of the network is a self-similar replicate of the whole. Hence this model relates components of structure and function that appeal to applied modelers. The WBE framework describes tree M with the following equation:

$$M = \mathrm{C}\rho\, D^{8/3} \qquad [1]$$

Where: C = a proportionality constant, and ρ = the specific gravity of the entire aboveground biomass. The scalar exponent, B_{WBE}, is fixed to 8/3 = 2.67 and specific gravity is referred as the total tree specific gravity (a weighted average of wood, bark, branches and leaves).

A second allometric model that physically describes tree M was preliminarily proposed by Návar (2010), as:

$$M = \mathrm{CV}\rho \qquad [2]$$

Where: V = total standing tree volume (m^3); ρ = the specific gravity of the entire tree aboveground biomass (Mg m^{-3}), and C = form factor for non-Euclidean objects.

Model [2] was derived from the classic mass physics equation. Bole volume is conventionally reported in forest inventories unlike branch volume that has to be computed using other independent methods. On the other side, the specific gravity values for entire trees are hardly known and the best guess is to assume that the bole wood specific gravity, ρ_w, conventionally measured in wood technology studies equals ρ. Tree boles have non standard idela shapes, therefore, the C form-factor accounts for this variability.

Empirical equations. Empirical allometric equations are statistically parameterized and they conventionally calculate tree M. Návar (2010) reported the most common empirical model of aboveground biomass estimation is the log-transformed equation [3]:

$$\mathrm{Ln}\,(M) = \mathrm{Ln}(a) \pm B\,\mathrm{Ln}\,(\mathrm{D}) \pm e_i = a\mathrm{D}^B + e_i \qquad [3]$$

Where: *a* and *B* are the scalar intercept (*a*) and exponent (*B*) of equation [3]; both parameters are usually calculated by least square techniques in log linear regression; e_i = the error. Note that the *a*–scalar intercept of model [3] = Cρ of model [1] and that B_{WBE} = 2.67. Also note that $B\mathrm{w} \neq B$.

Other new, modern empirical equations, which expand from equation [2] that contains the bole wood specific gravity and canopy height in addition to diameter at breast height were proposed by Chavé et al. (2005) for tropical forests as well as by Návar (2009a) for Mexico's northern temperate forests. A mathematical example of this model class reported by Chavé et al. (2005) is presented by model [4]

$$M = \left[(p_w \cdot a)D^b H^h\right] = \left[(p_w \cdot a)\cdot(D^2 H)^h\right] \qquad [4]$$

Semi-empirical models

The constant *B*-scalar exponent. Návar (2010) proposed that mean scalar coefficients close to the population means can be found in studies that compile empirical aboveground biomass equations. When diameter is measured at breast height, an average *B* value of 2.38 has been reported by most meta-analysis studies for model [5] (Návar et al., 2013). The *a*-scalar intercept value is a function of the bole wood specific gravity (Návar, 2010). With this assumption and a statistical function, the proposed reduced model is:

$$M = \left[a = f(p_w)\right]D^{b=2.38} \qquad [5]$$

or

$$M = [Cp_w]D^{b=2.38} \qquad [6]$$

The C value has been found to be 0.2457 for North American as well as for worldwide temperate tree species (Návar, 2010; Návar et al., 2013).

The shape-dimensional model. Návar (2010) developed the following preliminary semi-empirical non-destructive model using the shape-dimensional relations:

$$M = [a = f(B)]D^{B=(d+hB^*)} \qquad [7]$$

Where: *a*, *B* = the scalar intercept and exponent, respectively; d, h = the coefficients of the Schumacher and Hall (1933) volume equation; B* = the coefficient of the H-D log relationship. Similar equations for temperate species are found in Návar (2010).

Model [7] takes the advantage of the good relationship between scalar coefficients reported in the scientific literature (Zianis and Mencuccini, 2004).

Field data

In this report, major data sample details for the 39 harvested trees can be found in Návar-Cháidez (2009). The wood specific gravity values for these species were collected mainly from the list reported by Chavé et al. (2005). Inventory data for 390 plots with dimensions of (20 x 50 m) distributed at the ejidos Vado Hondo and San José Tiniaquis, Sinaloa, Mexico were available for fitting allometric equations.

Allometric equations and testing their performance

A total of eleven; two on site and nine off site allometric equations (seven empirical; two semi-empirical non-destructive; and two theoretical) were fitted to measured total aboveground biomass for all 39 Sinaloan tropical dry trees (Table 1).

The Návar's (2012) non-destructive method uses the H-D and V=H,D functions as well as an empirical equation to estimate the *a*-scalar intercept as a function of B. The West et al. (1999) model assumed that $\rho = \rho_w$ and C = proportionality constant. C coefficient values are available in the scientific literature (Návar, 2010) however they could not be found for tropical forests. Therefore the C coefficient value was estimated in three different forms: (a) by iteratively searching the C value; (b) by calculating a mean value with the equation; $C = M/(\rho_w D^{2.67})$; and (c) by statistically calculating it in non-linear regression. The physics equation also assumes that $\rho = \rho_w$. D = diameter at breast height, H = canopy height, BA = basal area, ρ_w = bole wood specific gravity; NA = not available. Note that the Návar (2010) and West et al. (1999) equations are similar in nature, with the exceptions noted above. E-C = empirical-conventional, E-U = empirical-updated, S-E-C = semi-empirical-conventional, S-E-U = semi-empirical-updated non-destructive, T = theoretical models, All = all forests since they are theoretical and semi-empirical models in nature.

Table 1. Allometric equations fitted to 39 destructively sampled trees of a Mexico's Sinaloan tropical dry forest community.

Researcher	Tropical Forest	Model	Equation
Brown (1997)	Dry	E-C	$M=34.47-8.0671D+0.6589D^2$
Brown (1997)	Dry	E-C	$M=\exp(-1.996+2.32Ln(D))$
Martínez et al. (1992)	Dry	E-C	$M=10^{(-0.5352+Log10(BA))}$
This report updated	Dry	E-U	$M=0.08479(\rho_w^{0.55255}D^{2.2435}H^{0.4773})$
This report conventional	Dry	E-C	$M=e(-2.409+0.952*Ln(D^2H))$
Chavé et al. (2005)	Dry	E-U	$M=0.112*(\rho_w D^2 H)^{0.916}$
Chavé et al. (2005)	Dry	E-U	$M=\rho_w*e(-0.67+1.78Ln(D)+0.207(ln(D))^2-0.028(Ln(D))^3)$
Návar (2010)	All	S-E-C	$M=(38.36*B^{-6.9045})D^{(B=d+hB^*)}$
Návar (2010)	All	S-E-U	$M=(-0.0094+0.2687\rho_w)D^{2.38}$
West et al. (1999)	All	T	$M=C\rho(D^{2.67})$
Návar et al., (2013)	All	T	$M=CV*\rho_w$

The equations of Brown (1997), Martínez-Yrizar (1992) and the second one developed for this report are conventional in nature, the updated equation developed for this report; the equations of Chavé et al. (2005), the equation of Návar (2010), the physics equation, and the West et al. (1999) equation, are improved, updated models since they contain other exogenous variables in addition to diameter at breast height. All these equations fitted the forest inventory data for all 390-0.10 ha plots to understand sources of inherent plot M variability as well.

Two error sources or precision level: a) the standard deviation and b) the standard deviation as percentage or coefficient of variation described the inherent aboveground biomass heterogeneity estimates by all equations. Equation [8] calculates the standard deviation, Sxe, across tree allometric equations:

$$Sxe = \sqrt{\frac{\sum_{i=1}^{n}\left(Mi - \overline{M}\right)^2}{n-p-1}} \qquad [8]$$

Where: Mi = mean aboveground biomass estimated by allometric equation i (Kg tree^{-1}), $\overline{M}$ = mean aboveground measured tree M (Kg tree^{-1}), and n = number of observations, p = number of parameters estimated for each allometric equation.

The standard deviation in percentage, Sx(%), was calculated as a function of the mean measured aboveground biomass figure of all allometric equations used by equation [9]:

$$Sx(\%) = \frac{\sqrt{\frac{\sum_{i=1}^{n}\left(Mi - \overline{M}\right)^2}{n-p-1}}}{\overline{M}} \cdot 100 \qquad [9]$$

Where: $\overline{M}$ = mean measured tree (Kg).

RESULTS

The updated modern tree M equation is: $M=0.08479(\rho_w^{0.55255}D^{2.2435}H^{0.4773})$; where: ρ_w = wood specific gravity; D = diameter at breast height (cm); H = top height (m). The standard error, Sx; the Sx(%), and the coefficient of determination, r^2, for this equation are 34.7 Kg; 40.62%, and 0.88, respectively. This improved equation recorded the best goodness of fit statistics of all 11 fitted models.

The eleven equations present different fitness degree and six of them predict individual tree M values, while the remaining five equations reports mean tree M estimates across D figures (Figure 1). Allometric equations with the exogenous variables diameter at breast height, the bole wood specific gravity and canopy height predict single M figures. They are individually graphed to better observe goodness of fit between tree M modeled and measured data. The conventional equations that contain only diameter at breast height provide mean tree M projections across the D range. Hence, they are reported in a single figure at the bottom of Figure 1.

The error as standard deviation and as standard deviation in percentage (coefficient of variation) show, as expected, that on site allometric equations provide the best goodness of fit statistics (r^2 = 0.88 and 0.78; and standard deviations (%) of 40.6% and 52.5%; for the updated and the conventional equations, respectively) (Table 2).

Youkhana and Idol (2011) reported similar results for *Leucaena leucocephala* Hawaian agroforestry trees; wehere these authors tested seven different off site allometric biomass equations and all they biased total tree M. Un-accounted sources of variation can be explained by a lack of on site bole wood specific gravity measured values that vary between trees of the same species (Parolin, 2002; Chavé et al., 2006; Silva-Arredondo and Návar, 2009) since mean values were acquired from Chavé's et al. (2006) lists.

The Chavé's et al. (2005) worldwide off site biomass equations perform not as good as the ones locally built up, as expected as well. Three other equations are worth discussing since they had fit statistics close to the on site conventional model. They are in descending order of fit: (a) the semi-empirical shape-dimensional, non-destructive equation proposed by Návar et al., (2013); (b) the physics equation (Návar, 2010); (c) the WBE theoretical model (West et al., 1999). All these equations reported coefficients of determination above 0.73 and standard deviations of less than 50.5 Kg tree^{-1}. All they have at least one theoretic component and the former one is conventional in nature.

The remaining allometric equations; the conventional (Brown, 1997; Martínez-Yrizar et al., 1992) and the modern, updated ones (Chavé et al., 2005; Návar, 2010) produce deviations > 52.5 Kg tree^{-1}, and coefficients of determination < 0.70. Two equations resulted in statistically different M estimates; the equation of Chavé et al. (2005) with an r^2 of 0.26 and a SD of 110 Kg tree^{-1}; and the semi-empirical non-destructive model that assumes a constant *B*-scalar exponent proposed by Návar et al., (2013) with an r^2 of 0.41 and a SD of 114 Kg tree^{-1}.

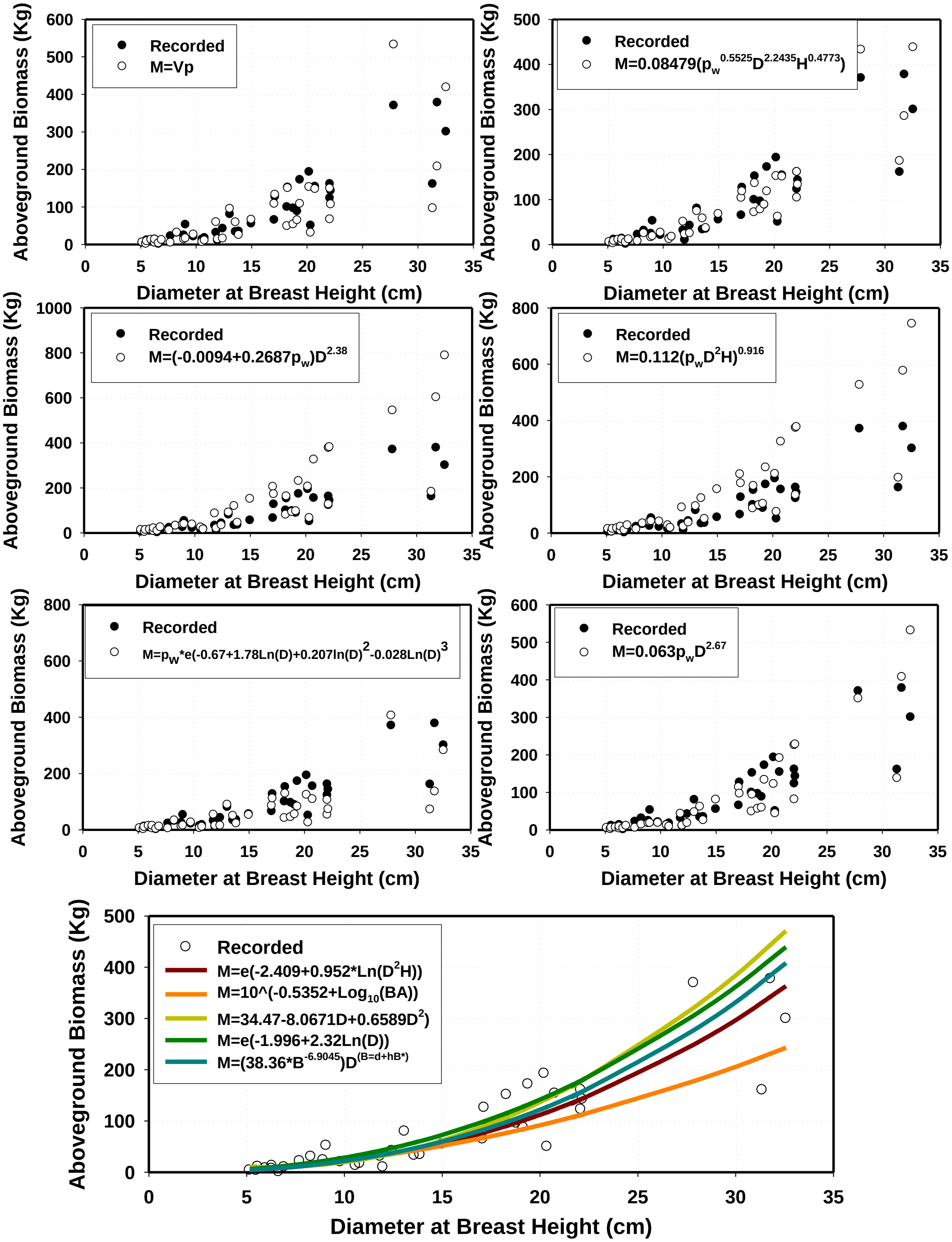

Figure 1. Measured and projected aboveground biomass values with eleven equations for Mexico's Sinaloan tropical dry trees.

Table 2. Fit statistics for aboveground tree biomass recorded and projected values by allometric equations developed on and off site for Mexico's Sinaloan tropical dry forests.

Researcher	Tropical Forest	Model Class	R^2	Standard Deviation	
				(Kg tree^{-1})	(%)
Brown (1997)	Dry	E-C	0.61	60.00	70.14
Brown (1997)	Dry	E-C	0.68	54.60	63.83
Martínez et al. (1992)	Dry	E-C	0.70	52.53	61.41
This report	Dry	E-U	0.88	34.75	40.62
This report	Dry	E-C	0.78	44.93	52.52
Chavé et al. (2005)	Dry	E-U	0.26	110.80	129.53
Chavé et al. (2005)	Dry	E-U	0.69	54.99	64.28
Návar (2010)	All	S-E-C	0.74	50.21	58.70
Návar (2010)	All	S-E-U	0.41	114.37	133.70
West et al. (1999)	All	T	0.73	50.16	58.64
Physics (Návar, 2010)	All	T	0.73	49.57	57.95
Mean values for all off site equations			0.62	66.35	77.58

All = All tropical forests.

Therefore, caution must be taken when selecting an off site biomass equation, since tree M deviations can under (Chavé et al. 2005) or overestimate (Chavé et al. 2005; Návar, 2010) tree M by close to one order of magnitude. That is, while mean measured tree M is 85.5 Kg tree^{-1}, the equations of Chavé et al. (2005) and Návar (2010) and Chavé et al. (2005) project 60.7 and 139.5 and 140.7 Kg tree^{-1}, respectively.

Mean plot M estimated by all eleven models are depicted in Figure 2. Confidence bounds show both Brown (1997) allometric equations project compatible plot M assessments with the on site improved and conventional allometric equations. Other off site allometric equations under (Martínez-Yrizar et al., 1992; Chavé et al., 2005) or over (Návar, 2010; Návar et al., 2013) estimate plot M by more than one order of magnitude. When selecting an equation to assess plot M, bear in mind the potential error and bias.

DISCUSSION

The most important source of error when estimating tree, plot or regional M is the choice of an off site allometric model (Chavé et al. 2004; 2005; Houghton, 2005a,b; Návar, 2010). In this report, it is shown that tree M can be biased by close to one order of magnitude in doing so for Mexico's Sinaloan tropical dry trees and forests. Using statistical backgrounds, Návar (2010) put forward the hypothesis that a mean tree M equation calculated with a combination of at least two independent equations could improve estimates beyond any single off site allometric model. Table 3 shows this hypothesis is correct for the case-studies described below. Data graphed for *Leucaena leucocephala* Hawaian agroforestry trees appears to improve goodness of fit statistics when pooling together all seven off site equations but they would still slightly bias tree M (Youkhana and Idol, 2011).

All goodness of fit statistics improves when combining more than one equation into a single one. For example, using mean values reported in Tables 2 and 3; the coefficient of determination increases on the average by 25% when combing more than two equations and it attains a value close to the empirical-updated equation developed for this study when combining all off site allometric equations. The standard deviation is reduced by 40% when mixing at least two allometric equations. Therefore, in the absence of local tree allometry, as it is the case for many dry forests, combined off site allometric equations would improve tree M assessments, consistent with the assumptions of the central limit theorem. Then, it is recommended to always bear in mind the worldwide, as well as the semi-empirical and theoretical allometric models available. Whenever it is possible, they should be locally calibrated.

A few single equations can be locally calibrated for further tree and plot M assessments for Sinaloan tropical dry forests. For example, the semi-empirical non-destructive, reduced equation that assumes a constant *B*-scalar exponent value improves its r^2 from 0.41 to 0.83 and reduces its standard deviation from 114.27 to 40.22 Kg tree^{-1} if the C coefficient (0.2457) is replaced by 0.135. Ketterings et al. (2001) reported a C value of 0.11 for Southeast Asian tropical trees. However, for plot M, the new C value that matches the on site and theoretical model has to have a C value of 0.175. The C=0.175 is consistent for both forest inventory data sources (San Fco Javier Tiniaquis n = 168 and Vado Hondo n=222). The former C value was calculated with a statistical program in non-linear regression. While other C coefficient estimates ($C = M/(\rho_w D^{2.38})$) or by trial and error produce better tree M estimates, the statistical calculation resulted in the best tree M approximations and warrant fit statistics close to the empirical-updated on site allometric equation fitted to this data set. The single Chavé et al. (2005) model (i.e., $M=0.112*(\rho_w D^2 H)^{0.916}$) recover the r^2 from 0.69 to 0.82 if the model parameters are replaced (e.g., $M=0.1847*(\rho_w D^2 H)^{0.8881}$). Plot M assessments also match the on site allometric equation with this new set of parameters. The second Chavé's et al. (2005) equation (e.g., $M=\rho_w*e(-0.67 + 1.78Ln(D) + 0.207ln(D)^2 - 0.028Ln(D)^3)$ can only be improved by fitting the single exponential function (e.g., $M = \rho_w*exp(-1.92899+2.4226*Ln(D))$) with an r^2 value of 0.83 instead of the 0.29 previously calculated. Using this new set of parameters, plot M evaluations are also matched with those projected by the on site improved allometric model.

The on site updated equation provides the best goodness of fit statistics in contrast to the locally calibrated individual allometric equations or any combination of off site equations. Therefore, it is highly recommended to destructively harvest at least a few trees for developing or fitting theoretical tree allometry. For tropical forests, I strongly recommend harvesting a few trees in the range of the highest diameters and the dominant species found in these forests since the largest variation is encountered in the right hand side of the allometric equation.

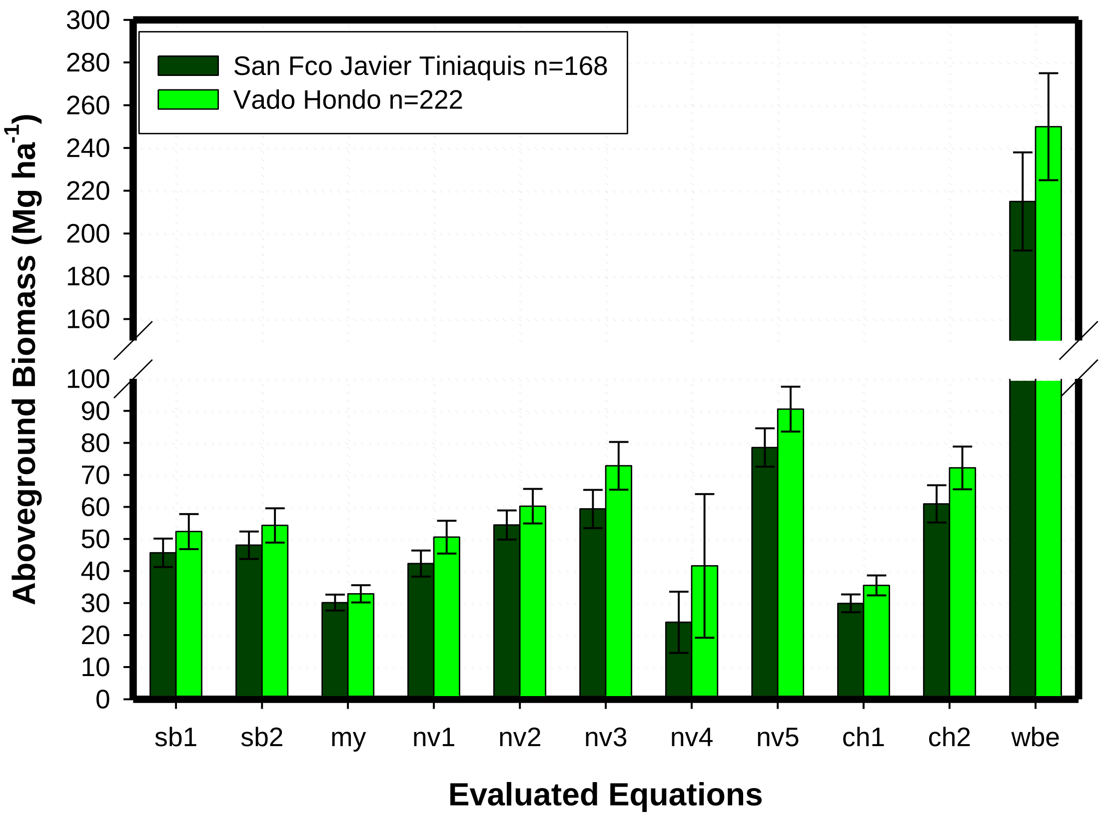

Figure 2. Forest inventory plot aboveground values calculated with 11 biomass equations for Mexico's Sinaloan tropical dry plots (sb=Brown, 1997; nv = Návar; my = Martinez-Yrizar et al., 1992; ch = Chavé et al., 2005; wbe= West et al., 1999).

Table 3. Testing the hypothesis that mean values derived from equation samples provide better fit statistics than single sample equations.

Allometric Equation Class	Average Statistics			Author
	R^2	Standard Deviation		
		(Kg tree^{-1})	(%)	
Conventional	0.77	47.80	55.8	Brown (1997); Martínez et al. (1992)
Conventional-Updated	0.74	51.08	59.7	Chavé et al. (2005)
Semi-empirical, non-destructive	0.75	49.78	58.2	Návar (2010)
Conventional, semi-empirical and theoretical	0.84	40.47	47.3	All off site equations
Mean Values	0.78	47.28	55.3	

CONCLUSIONS

This report aimed at developing an updated on site allometric equation and understanding inherent sources of tree and plot M heterogeneity when contrasting on and off site biomass allometric equations for Mexico's Sinaloan tropical dry trees and forests. The modern equation improved tree M estimates, but part of the un-explained variation could be accounted by the bole wood specific gravity that can be measured later in the field with higher degree of precision using non-conventional approaches that are being under development. In the meantime, a list of wood specific gravity values has been reported for tropical dry species that is available for applying theoretical, semi-empirical and a few empirical equations. The off site equations can produce tree and plot M estimates with deviations as large as one order of magnitude in contrast to the on site improved allometric equation. Therefore, caution must be taken when fitting tree allometry for trees and forests deprived of these techniques in tropical dry ecosystems. Individual equations could be calibrated on site by fitting new parameters with the same tree M data. In the absence of destructively harvested trees, a combination of off site biomass equations would increase tree and plot M precision in contrast to fitting single recommended equation.

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