Tropical and Subtropical Agroecosystems

MODELING OF MILK PRODUCTION AND COMPOSITION CURVES IN A HERD OF F_1 ALPINE x NUBIAN GOATS IN SAN LUIS POTOSÍ, MÉXICO

[MODELACIÓN DE CURVAS DE PRODUCCIÓN Y COMPOSICIÓN DE LA LECHE EN UN HATO DE CABRAS F_1 ALPINA x NUBIA EN SAN LUIS POTOSÍ, MÉXICO]

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SUMMARY

Goat milk production is a major source of income for farmers in arid and semiarid regions of México. However, in Mexico there is limited information on the characterization of milk production curves and its components. The aim of this study was to characterize the lactation curves and their components (total solids, fat, protein) in F_1 (Alpine x Nubian) goats of the Potosino highlands, using three mathematical models. Monophasic, diphasic and incomplete gamma models were evaluated, considering as adjustment criteria the mean square error, pseudo R^2, Akaike information criterion, Bayesian information criterion and -2 log likelihood. To characterize the milk production curve, the diphasic model was the most consistent in all the criteria for adjustment, so it was more useful than the monophasic and incomplete gamma models. For the curves of the milk components, the best adjustment corresponded to the incomplete gamma model.

Key words: Mathematical functions; lactation curves; milk components; goats.

RESUMEN

La producción de leche de cabra constituye una fuente muy importante de ingresos para los productores en las regiones áridas y semiáridas de México. Sin embargo, hay información limitada en México sobre caracterización de las curvas de producción de la leche y sus componentes. El objetivo de este estudio fue caracterizar las curvas de lactancia y sus componentes (sólidos totales, grasa, proteína) en cabras F_1 Alpina x Nubia del altiplano Potosino, utilizando tres modelos matemáticos. Se evaluaron los modelos monofásico, difásico y gamma incompleta, considerando como criterios de ajuste el cuadrado medio del error, pseudo R^2, criterio de información de Akaike, criterio de información Bayesiano y -2 veces el logaritmo de la verosimilitud. Para la caracterización de la curva de producción de leche el modelo difásico fue el más consistente en todos los criterios de ajuste, por lo que resultó más útil que los modelos monofásico y gamma incompleta. Para las curvas de los componentes de la leche, el mejor ajuste correspondió al modelo gamma incompleto.

Palabras claves: Funciones matemáticas; curvas de lactancia; componentes de la leche; cabras.

INTRODUCTION

In Mexico, production of goat´s milk represents an important source of income in the arid zones and most of the milk produced (70%) comes from extensive systems in this area (Mellado et al., 2004). The graphical representation of milk production over time is a lactation curve (Sherchand et al., 1995). The common behavior of the curve is that the production increases at the beginning up to a maximum (production peak), this peak is maintained, and decreased gradually until the end of lactation (Wood, 1967). Knowledge of a lactation curve is a valuable tool because it can be used for different aspects of management, such as food (Sherchand et al., 1955) and genetic improvement (Weigel et al., 1992). This curve has been modeled with different mathematical equations which, despite of difference in its formulation, all generate a similar graphic (Tozer and Huffaker, 1999). The usefulness of each of these depends on how well describe o mimic the biologic process of lactation (Olori et al., 1999), evaluating them through certain criteria of adjustment, as the mean square error and pseudo-R^2 (Schabenberger, 2008), Akaike information criterion (Akaike, 1974), Bayesian information criterion (Sawas, 1978) and -2 log likelihood (Juneja et al., 2007). The parameters of the equations, as far as possible, must have a biological meaning (Chang et al., 2001). Among the most commonly used functions are the incomplete gamma, inverse polynomial, segmented polynomial and multi-phasic function. The latter divides the curve at different stages providing useful information of the curve characteristics, including a scale (initial and maximum production) and shape (time of maximum production and persistence) (Gipson and Grossman, 1990), and provides a better adjustment than the commonly used incomplete gamma function (Grossman and Koops, 1988), as well as allows to develop adjustment factors to correct environmental effects (De Boer et al., 1989). Besides the curve for milk production it is important to adjust the curves for components, because the quality and price of milk usually depend on the content and characteristics of these components (Lombard, 2006). Despite the importance that goat milk production has in arid and semiarid regions of Mexico a curve for milk production or its components for this kind of goats has not been characterized, that is why this study was carried out to characterize lactation curves and components of milk in a herd of F_1 Alpine x Nubian goats in San Luis Potosí, Mexico.

MATERIAL AND METHODS

Location and animals

The study was conducted at the Goat Unit, Department of Animal Production, Faculty of Agronomy, Universidad Autónoma de San Luis Potosí (UASLP), with coordinates 22°14’10’’ latitude North and 100°53’10’’ longitude West, at an altitude of 1833 m, with an average annual rainfall of 335mm. The dry desert climate is cold, with an average annual temperature of 18 °C (7.5 °C minimum and 35 °C maximum) (García, 1973). Data from the production of 18 F_1 Alpine x Nubian goats of second parity were utilized which had given birth to only one offspring. The goats were kept stabling in pens equipped with feeders and drinkers. The goats were fed during the last third of gestation and the sampling period with 2 kg of dry matter contributed by 3.7 kg of green alfalfa and 1 kg on a fresh basis of commercial concentrate with 16% crude protein and 2.4 Mcal; concentrate was provided in equal parts in the morning and afternoon, while the forage was provided only in the morning. The goats were milked once daily morning for 14 weeks. Data were obtained on the production and milk composition (total solids, fat and protein). Milk samples were collected in glass bottles the same day that production was measured, and samples were analyzed every 15 days to determine their composition (total solids, fat and protein); the analysis of the samples was carried out by using The Milko Scan™ Minor in the Faculty of Agronomy, UASLP.

Statistical Analysis

Descriptive analysis. Firstly, mean, standard deviation, coefficient of variation (CV), maximum and minimum production for each of the variables under study were calculated; per week for milk production and biweekly for total solids, protein and fat in milk.

Models used. Monophasic (Sherchand et al., 1995), diphasic (Gipson and Grossman, 1989) and incomplete gamma (Wood, 1967) functions were evaluated, which are shown below:

Monophasic function:

$$Y_t = a_i\, b_i \left[1 - \tanh^2\left(b_i\left(t - c_i\right)\right)\right]$$

Diphasic function:

$$Y_t = \sum_{i=1}^{2} \left\{a_i\, b_i \left[1 - \tanh^2\left(b_i\left(t - c_i\right)\right)\right]\right\}$$

Where: Y_t = milk production at a time *t*; a_i = average total production by phase (kg); b_i = production rate of a_i (per day) by each phase i; c_i = time of peak production (days) for each phase i; ***t*** = time of lactation (days); $\tanh$ = hyperbolic function of the tangent.

Incomplete gamma function:

$$Y_t = at^b\, e^{-ct}$$

Where *Y* is the milk production in the time *t*, which is the time of lactation (days), *a* is the parameter that represents a scale factor or milk production at the beginning of lactation, *b* and *–c* represent the limit of the slope of the curve before and after peak lactation, respectively, while *e* is the base of the natural logarithms.

Parameter estimation and model comparison. The estimation of parameters and adjustment criteria were obtained using the NLMIXED procedure of the statistical SAS® package (SAS, 2002). For comparison of the models, as criteria of adjustment: mean square error (MSE), pseudo–R^2, Akaike information criterion (AIC), Bayesian information criterion (BIC) and -2 log likelihood were considered. The lowest value of these statistical values, except for pseudo-R^2, is considered as the best adjustment, the reason why the best model was selected as one that was more consistent in these criteria.

AIC (Akaike, 1974):

$$AIC = n \ln\left(\frac{SSE}{n}\right) + 2p$$

BIC (Sawa, 1978):

$$BIC = n \ln\left(\frac{SSE}{n}\right) + 2\left(p+2\right)q - 2q^2$$

Where n is the number of observations; SSE is the sum of squares in the model; p is the number of parameters in the model; $q = n\hat{\sigma}^2 / SSE$ and $\hat{\sigma}^2$ is the estimated error of variance of the adjusted model.

Pseudo-R^2 defined by:

Pseudo-R^2 = 1-SC(Residual)/SC(Total $_{Corrected}$)

Lactation curves and characteristics. Curves were determined with the model that had the best adjustment. Characteristics were calculated by means of $\boldsymbol{a_i b_i}$, $\boldsymbol{c_i}$ and $\boldsymbol{2/b_i}$ for the monophasic and diphasic functions (Gipson and Grossman, 1990); and with $\boldsymbol{a(b/c)e^{-b}}$, $\boldsymbol{b/c}$ y –($\boldsymbol{(b+1)\ln(c)}$) for the incomplete gamma function (Rekik and Ben, 2004); for the production at peak, time of occurrence of the peak, and persistence respectively.

RESULTS AND DISCUSSION

Descriptive analysis of data

Milk production. Milk production showed an atypical behavior, presenting two peaks of production (Figure 1), one at week 2 (1133.8±236.4 ml) and other at week 12 (1152.2±232.5 ml). This behavior is attributed to a short period (4-5 days) in which there were problems with availability of the concentrate food. The most critical period occurred at week 8 (845.2 +172.8 ml).

Milk components. The average total solids content was 12.12%, which is identical to that reported by Diaz et al. (2004) in goats of the same genotype, much lower than that reported by Ayala-Oseguera and Armendariz (2003) and by Sanz et al. (2006) in dairy goat breeds in intensive conditions in general, and higher than that obtained by Iaschi et al. (2004) in Boer and Australian goats under extensive conditions; these differences, apart from attributed to different genotypes, also reflect the production system. The total solid content in the first biweekly was higher than that found by Diaz et al. (2004) (13.46% *vs* 12.5%), but in the third fortnight averages were identical (11.7%). The average protein content was 3.29%, similar to that reported by Vega and Leon et al. (2004) in Alpine goats, but lower than the values obtained by Diaz et al. (2004) and Iaschi et al (2004). The highest percentage of protein was observed in the first week (3.51± 0.32%) and lower in weeks three and four (3.13 ± 0.51 and 3.12 ± 0.29%, respectively). The average fat content was 3.40%, similar to that reported by Diaz et al. (2004), but lower than those reported by Ayala-Oseguera and Armendariz (2003) and Sanz et al. (2006), which averaged 4.25%. Fat percentage in the first biweekly was the highest (4.08%), and higher than that obtained by Diaz et al. (2004) (3.43%); however, in the third biweekly was lower (3.22 *vs* 3.52%). The lower fat content was obtained in the fourth fortnight (2.74±0.81%). Pletcher and Jaffrézic (2002) have pointed out that the genotype is the main factor that explains the differences in the milk components, mainly in concerning fat and protein, according to Kala and Prakash (1990) by using Barbari and Jamunapari goats, as well as Valencia et al. (2005) with Saanen goats, which is attributed mainly to the polymorphism level in genes α_{s1}-casein and α_{s2}-casein (Marletta et al., 2003; Moioli et al., 2006).

Estimation of parameters, comparison of models and curves

Milk production. The estimated parameters and production curves for the different used models are shown in Table 1 and Figure 1; respectively. In Figure 1 an atypical curve is perceived; as Gipson and Grossman (1989) mentioned in lactations of dairy goats in general, as well as Tozer and Huffaker (1999) in Australian Holstein cows, and Faro and Albuquerque (1999) in Caracu cows of Brazil, the incomplete gamma function overestimates the milk production at the beginning of lactation.

According to adjustment criteria considered, the model that best adjusted (Table 2) was the diphasic model designed to model multiple peaks (Weigel et al., 1992), and by dividing the lactation curve in two phases (Phase 1 and Phase 2, Fig. 2), provides a better idea of the food management to maintain the maximum production for as long as possible. In addition, according to results obtained by Rekaya et al. (2001) and Faro and Albuquerque (2002) in dairy Holstein and Caracu cows, respectively, in case of an atypical curve the incomplete gamma does not adjust to data. These results coincide with those obtained by Scott et al. (1996) and Sherchand et al. (1995) who working with lactations of Holstein cows recognized the diphasic model as the best simulator of lactation, in comparison to models like incomplete gamma, inverse polynomial, and monophasic model; however, based on the adjustment criteria, do not coincide that the monophasic model is higher than the incomplete gamma model, and that it is equal to the diaphasic model. These results confirm what Gipson and Grossman (1990) also found in their revision on dairy goats in general, Macedo et al. (2001) in Saanen goats, and Gonçalves et al. (2002) in Holstein cows.

Components of milk. According to adjustment criteria, the model that better described the content of total solids, protein and fat in milk was the incomplete gamma (Table 3).

Table 1. Estimates of parameters for the curve of milk production of F_1 Alpine x Nubian goats.

Model	Parameters			Probability		
	a	*b*	*c*	*a*	*b*	*C*
Monophasic	165593.00	0.006	42.791	<0.0001	<0.0001	0.2963
Diphasic						
Phase 1	4947.00	0.208	2.666	<0.0001	<0.0001	<0.0001
Phase 2	5628.00	0.192	12.683	0.0002	<0.0001	<0.0001
Incomplete gamma	1033.45	-0.074	-0.017	<0.0001	0.2037	0.1212

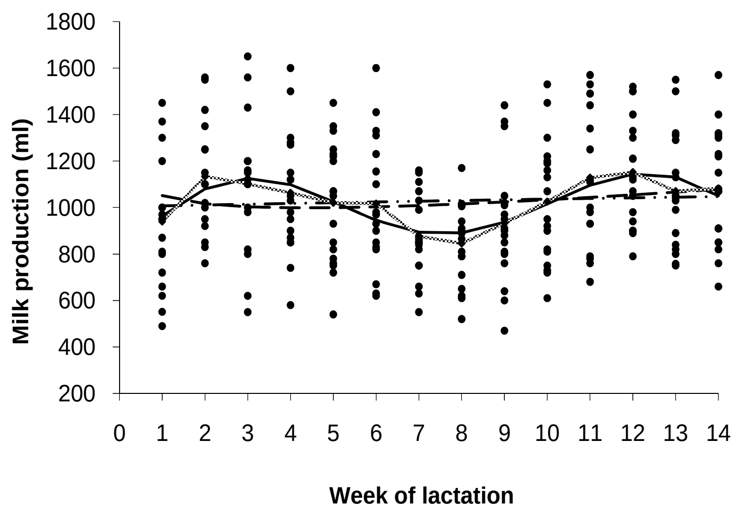


Figure 1. Observed milk production (•), average (┈+┈) and predicted through the diphasic (▬), monophasic (-··) and incomplete gamma functions (--) of F_1 Alpine x Nubian goats.

Table 2. Adjustment criteria utilized for the curve of milk production.

Model	Criterion				
	MSE	Pseudo–R^2	AIC	BIC	-2 log likelihood
Monophasic	12003.6	0.991	3529.1	3539.6	3523.1
Diphasic	2241.6	0.998	3505.6	3526.8	3493.6
Incomplete Gamma	11579.9	0.991	3527.2	3537.8	3521.2

MSE: mean square error, AIC: Akaike information criterion, BIC: Bayesian information criterion. With the exception of the first criterion, a lower value means better adjustment.

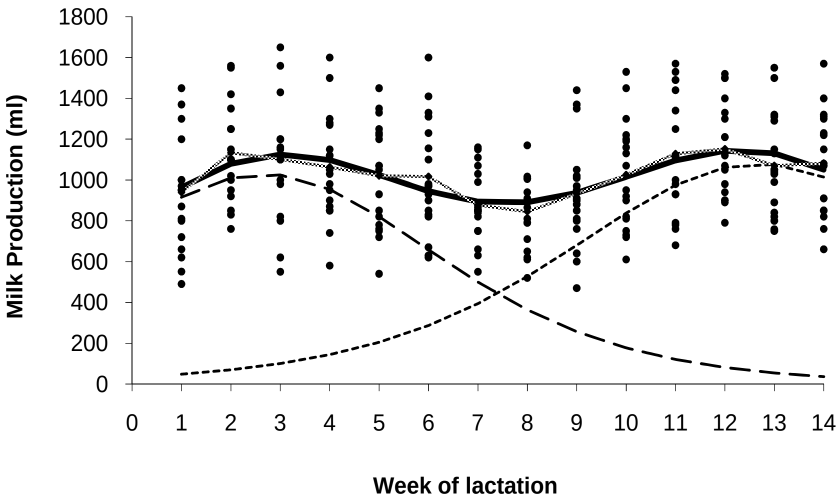


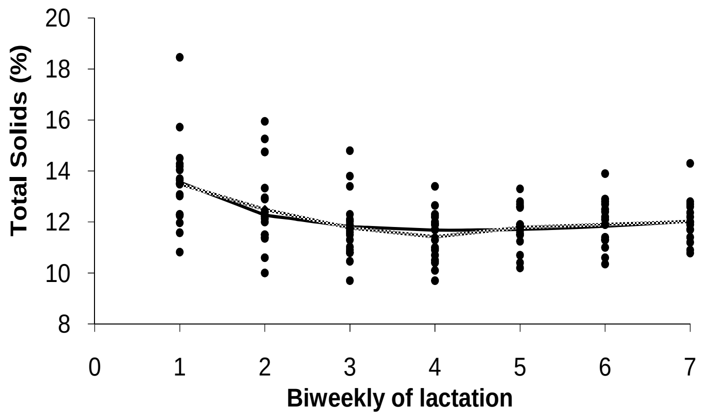
Figure 2. Observed milk production (●), average (┈+┈) and predicted through the diphasic function [general (━), Phase 1 (━ ━) and Phase 2 (▪ ▪)] of F_1 Alpine x Nubian goats.

Table 3. Criteria of adjustment used for the curve of the milk components of F_1 Alpine x Nubian goats.

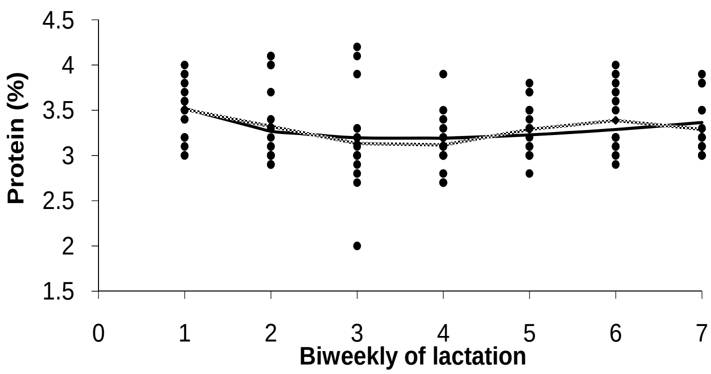
Model	Criterion				
	MSE	Pseudo–R^2	AIC	BIC	-2 log likelihood
Total solids:					
Monophasic	0.4101	0.986	426.3	434.8	420.3
Diphasic	0.4101	0.998	445.6	462.6	433.6
Incomplete gamma	0.0330	0.999	402.0	410.5	396.0
Protein :					
Monophasic	0.1484	0.848	121.1	129.6	115.1
Diphasic	0.1484	0.848	111.8	128.8	99.8
Incomplete gamma	0.1372	0.860	110.3	118.9	104.3
Fat:					
Monophasic	1.1153	0.886	394.0	402.5	388.0
Diphasic	1.1152	0.886	399.5	416.5	387.5
Incomplete gamma	1.0033	0.897	386.6	395.1	380.6

MSE: mean square error, AIC: Akaike information criterion, BIC: Bayesian information criterion. With the exception of the first criterion, a lower value means better adjustment.

The total solid content at the initiation of lactation was high and continued a downward trend during the remainder of lactation (Figure 3a), behavior is consistent with the results obtained by Diaz et al. (2004) in Alpine x Nubian goats and Greyling et al. (2004) with Boer and native goats of Africa. The protein content did not show the typical tendency that Lennox et al. (1993) and Lombaard (2006) described in their research on modeling; in the 1st biweekly a high percentage was observed, in the 2nd biweekly decreased and reached the minimum in the 3rd and 4th biweeklies; in the 6th biweekly again raised and, contrary to the typical tendency, once again decreased in the 7th biweekly (Figure 3b). The fat content had a typical trend (Figure 3c), not exactly what happened with protein, an effect attributable to lack of food for 4-5 days (a fact noted above), and that is undoubtedly a factor modifying milk composition, according to the findings by Min et al. (2005) in dairy goats.



(a)



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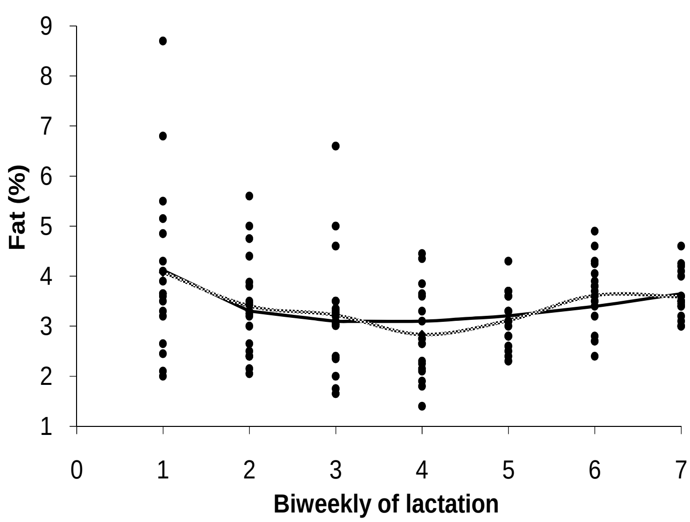

(c)

Figure 3. Observed content (•), average (···+···) and predicted through the incomplete gamma function (━) of the milk components [total solids (a), protein (b) and fat (c)].

Characteristics of lactation

Milk production at peak was 1029.97 and 1080.57 ml for the first and second phase, respectively. These maximum productions were presented at the 27 and 127 days, at the first and second phase, respectively. (Table 4); values lower than those found by Gipson and Grossman (1989) (Phase 1=day 37 and Phase 2 = day 113) and Montaldo et al. (1997) (day 56 of lactation) in dairy goats of European origin; but similar to that found by Ruvuna et al. (1995) (day 28 of lactation) in crosses of African with European breeds. Remember that the genotype studied here is a Alpine x Nubian cross of second parity, and the values vary due to the genotype (Gonçalvez et al., 2002) and the number of parity (Epaphras et al., 2004; Miranda and Schnitkey, 1995), are factors influencing the shape and characteristics of the lactation curve. Duration of each phase was of 9.61 and 10.42 days (Phases 1 and 2; respectively, and there is a considerable difference with the values obtained by Gipson and Grossman (1989) (Nubian =274 and 952 d; Alpine = 179 and 606 d for the Phases 1 and 2, respectively). This difference is mainly attributed to the fact that these authors included lactations of 305 days, which did not happen in this study. There are few reports in literature with respect to curves of milk components for goat herds, besides only obtained values by the incomplete gamma function are reported. For total solids, the production at peak was 11.67% and it occurred at 30.10 days of lactation, with a persistence of 16.37 days. The highest protein production was 3.18%, occurring at day 24.69 of lactation, with a persistence of 16.98 days.

For the fat content, production at peak was observed at day 24.38 with 3.08% with a persistence of 5.95 days (Table 4). These estimators of milk components are slightly lower than those reported by Diaz et al. (2004) (3.42, 3.57 y 12.12 %, for fat, protein and solids totals; respectively) in goats of the same genotype.

CONCLUSION

For its lower values in the mean square error, Akaike information criterion, Bayesian information criterion, and -2 log likelihood, as well as its higher value of the pseudo-R^2, the diphasic model was the best one that characterized the curve of milk production of F_1 Alpine x Nubian goats. As to the characterization of the curves of total solids, fat, and protein, and based on the same adjustment criteria, the incomplete gamma function was superior to monophasic and diphasic models.

Table 4. Characteristics of lactation of F_1 Alpine x Nubian goats.

Model	Characteristic		
	Production at peak (ml)	Time at peak (days)	Persistence (days)
Milk Production:			
Diphasic			
Fase 1	1029.97	27	9.61
Fase 2	1080.57	127	10.42
Total solids:			
Incomplete gamma	11.67	30.10	16.37
Protein			
Incomplete gamma	3.18	24.69	16.98
Fat:			
Incomplete gamma	3.08	24.38	5.95

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